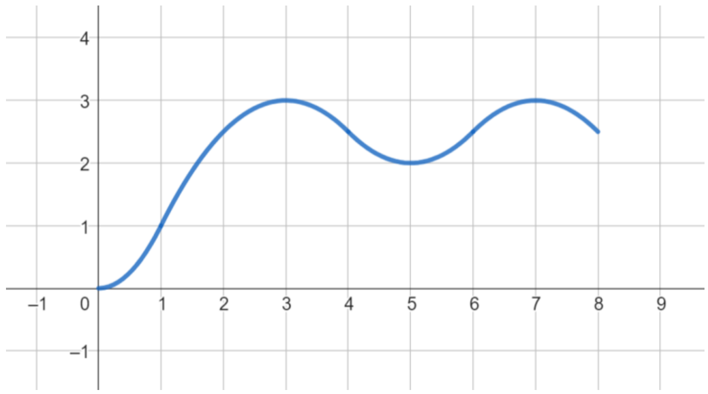
AP Precalculus Midterm Review

These problems are good practice for the AP Precalculus Midterm, but do not expect the midterm to be these exact questions just with different numbers. Use these problems to remember all the topics we have discussed this year. Then, practice those topics that you think need reinforcement. If you find yourself struggling, freak out enough to motivate you to study, but do not freak out so much that you become incapacitated. The midterm is just a tool to check how we're doing. Nothing more.

1. Consider the polynomial $g(x)$ defined on $0 \le x \le 8$ graphed below.



   a. What is the absolute maximum value of $g(x)$?
   b. What is the global minimum value of $g(x)$?
   c. At what x-value(s) does $g(x)$ have local minima?
   d. At what x-value(s) does $g(x)$ have relative maxima?

x	0	2	4
$f(x)$	?	15	20

2. The table above shows values of a function f at selected values of x. Which of the following values of $f(0)$ would be consistent with the claim that $f(x)$ is increasing and concave down over its entire domain?
   (A) $f(0) = 8$
   (B) $f(0) = 10$
   (C) $f(0) = 12$
   (D) $f(0) = 18$

3. If $f(x)$ is a polynomial function, then each of the following describes an impossible scenario. Determine why each scenario is impossible.
   a. $f(x)$ is a quartic function, the leading coefficient of f is -8, $2 - i$ is a zero of f, the graph of $f(x)$ has three x-intercepts.
   b. The average rate of change of $f(x)$ over the interval $[-4,4]$ is $\frac{1}{2}$, $f(4) = -3$, $f(x)$ does not have a global minimum, $f(x)$ is an odd function

4. $f(x)$ is a rational function of the form $f(x) = \frac{n(x)}{d(x)}$ where $n(x)$ and $d(x)$ are polynomial functions. Explain why the following scenario is impossible: the domain of $f(x)$ is $(-\infty, -1) \cup (-1, \infty)$, $d(x)$ is cubic, $n(x)$ is quadratic, $\lim_{x \to -\infty} f(x) = -\infty$

For each of #5 and #6, find the equation of all vertical asymptotes, horizontal asymptotes, slant asymptotes, holes, x-intercepts, and y-intercepts. Then, using limit notation, express the end behavior of f as x decreases without bound.

5. $f(x) = \frac{6x^2+10x+4}{3x^2+7x+2}$

6. $f(x) = \frac{(2x^2-x-3)(x^2+2x+1)}{-x^2+5x+6}$

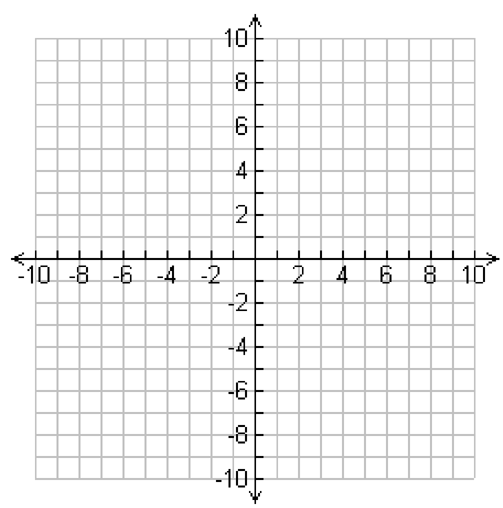
7. Find the equation of the slant asymptote of $y = \frac{-3x^3+1}{x^2+4}$.

8. Solve $\frac{(x-1)(x+3)^2}{(x-5)} \geq 0$

9. $f(x)$ is a polynomial function. Over the interval $0 \leq x \leq 6$, the average rate of change of $f(x)$ is 1. Over the interval $6 \leq x \leq 10$, the average rate of change of $f(x)$ is $\frac{7}{3}$. Which of the following is possible?
   (A) The rate of change of $f(x)$ over the interval $[0,10]$ is negative
   (B) $f(x)$ has an absolute maximum value at $x = 6$
   (C) $f(x)$ is decreasing over the interval $(6,10)$
   (D) $f(x)$ is concave up over the interval $(0,10)$

10. Which one of the following is an even function?
   (A) $f(x) = 2x^2 - 8x + 6$
   (B) $f(x) = 9x^4 - 1$
   (C) $f(x) = 4x - 1$
   (D) $f(x) = 2x^3 + 2$

11. On the graph provided to the right, sketch a polynomial function $f(x)$ for which the following are true:
   - $f(x)$ is a cubic function
   - $f(x)$ is decreasing and concave up on $(-\infty, -5)$
   - $f(x)$ is increasing and concave up on $(-5, -3)$
   - $f(x)$ is increasing and concave down on $(-3, -1)$
   - $-5 + \frac{1}{2}i$ is a zero of the function

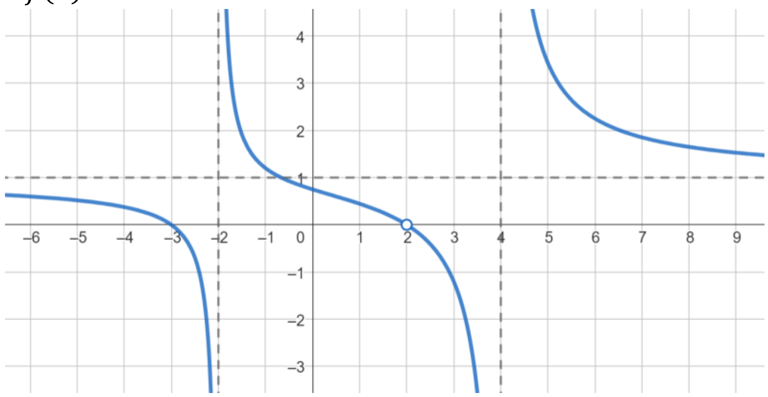


12. The table below shows values of the function $f(x)$

x	-4	-2	0	2	4	6
$f(x)$	-2	-4	-3	1	k	18

   a. Find the average rate of change of $f(x)$ over the interval $[-2,2]$.
   b. If $f(x)$ is quadratic, find the value of k.

13. Consider the graph of $y = f(x)$ shown below.



a. At which of the following x-values does $f(x)$ seem to have a point of inflection?

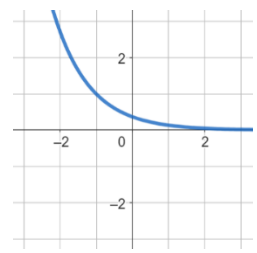
(A) -1.5 (B) 0.5

(C) 3 (D) 4

b. Find $\lim_{x \to -2^+} f(x)$

c. Find $\lim_{x \to 2} f(x)$

The graph of $g(x)$ is shown to the right. $g(x)$ is positive, decreasing, and concave up over its entire domain. Questions #14-16 give various transformations of this function. Match the number of each transformed function with the letter of the matching description.



14. $-g(x)$

15. $g(-x)$

16. $-g(-x)$

A) The rate of change of this negative function is negative and decreasing.

B) This negative function is increasing at a decreasing rate.

C) This function is increasing and concave up. Over its domain, the function is always positive.

17. Let $g(x) = f(-x)$. If $\lim_{x \to 3^+} f(x) = \infty$, then which of the following must be true?

(A) $\lim_{x \to -3^+} g(x) = \infty$

(B) $\lim_{x \to -3^-} g(x) = -\infty$

(C) $\lim_{x \to -3^+} g(x) = -\infty$

(D) $\lim_{x \to -3^-} g(x) = \infty$

18. The graph of $y = f(x)$ is tangent to the x-axis at two different x-values. $f(x)$ has three distinct real zeros. $f(-1 + 3i) = 0$. What is the least possible degree of f?

19. The rational function r is given by $r(x) = \frac{(x+1)(x^2+2x+5)}{(x-1)(x^2-4x-5)}$. How many vertical asymptotes, holes, and x-intercepts does this function have?

20. The polynomial p is of degree 5 with leading coefficient -2. One of the zeros of p is $2+3i$. Decide if each of the following statements must be true, might be true, or cannot be true.
    a. $2-3i$ is a zero of p
    b. $-2+3i$ is a zero of p
    c. $\lim_{x\to-\infty} p(x) = \lim_{x\to\infty} p(x)$
    d. p has 2 distinct real zeros
    e. The graph of $\frac{1}{p(x)}$ have four vertical asymptotes.

21. Over the 1900s, humans got taller. In 1915 ($t = 15$), the mean height of women was 60 inches. In 1990 ($t = 90$), the mean height of women was 63 inches. (Note: if these heights sound particularly short to you, keep in mind that they are a global average. Americans tend to be above the global average in height and weight.)

    Let $H(t)$ be a model that predicts the height of women, in inches, t-years after 1900.

    a. Use the given data to find the average rate of change of the mean height of women, in inches per year, from $t = 15$ to $t = 90$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer. (you can use a calculator/Desmos if that is helpful for the computation, but your work should show what sum/difference/product/quotient you evaluated with the calculator)

    b. Use the average rate of change found in part a to estimate the mean height of women, in inches, in 1945 ($t = 45$). Show the work that leads to your answer.

    c. If $H(t)$ is a concave down function, explain why the answer from part b must underestimate the value of $H(45)$.

22. For each angle, state the quadrant in which the terminal ray lies, state the reference angle, and state two coterminal angles (one positive, and one negative).
    a. $-\frac{6\pi}{5}$
    b. $\frac{7\pi}{4}$
    c. $\frac{7\pi}{3}$

23. Evaluate each of the following
    a. $\sin\left(\frac{3\pi}{4}\right)$
    b. $\cos\left(\frac{\pi}{2}\right)$
    c. $\tan\left(\frac{7\pi}{4}\right)$
    d. $\sec\left(\frac{4\pi}{3}\right)$
    e. $\csc(\pi)$
    f. $\cot\left(-\frac{\pi}{6}\right)$
    g. $\sin\left(\frac{13\pi}{2}\right)$
    h. $\cos\left(\frac{8\pi}{3}\right)$
    i. $\tan(0)$

24. When graphed in standard position, the terminal ray of θ passes through the point $(5,-12)$. Find $\sin\theta$, $\sec\theta$, and $\tan\theta$.

25. In a circle of radius 24, a central angle of θ subtends a minor arc of length 30. Find the value of θ.

26. Which of the following expresses all values of θ for which $\sec\theta$ is undefined?
    (A) $\frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$
    (B) $\frac{\pi}{2} + \pi k, k \in \mathbb{Z}$
    (C) $2\pi k, k \in \mathbb{Z}$
    (D) $\pi k, k \in \mathbb{Z}$

27. Which one of the following is equivalent to $\frac{1}{\sec\theta - \sin^2\theta\sec\theta}$?
   (A) $\sin\theta$
   (B) $\cos\theta$
   (C) $\sec\theta$
   (D) $\csc\theta$

28. Which one of the following is equivalent to $\frac{\csc\theta(\sec^2\theta - 1)}{\sec\theta}$?
   (A) $\sec\theta$
   (B) $\csc\theta$
   (C) $\tan\theta$
   (D) $\cot\theta$

29. Find an expression for all solutions to the following trigonometric equation.
$$2\sin^2\theta - 3\sin\theta - 2 = 0$$
   (A) $\theta = \frac{\pi}{6} + \pi k$, where k is any integer
   (B) $\theta = \frac{7\pi}{6} + 2\pi k$ or $\theta = \frac{11\pi}{6} + 2\pi k$, where k is any integer
   (C) $\theta = \frac{2\pi}{3} + 2\pi k$ or $\theta = \frac{4\pi}{3} + 2\pi k$, where k is any integer
   (D) $\theta = \frac{\pi}{3}k$, where k is any integer

30. Find all solutions in the interval $0 \le \theta < 2\pi$
$$3\cos\theta - 2\sec\theta = -1$$
   (A) $\theta = \pi$, $\theta = \pi + \arccos\left(\frac{2}{3}\right)$, $\theta = 2\pi - \arccos\left(\frac{2}{3}\right)$
   (B) $\theta = \pi$, $\theta = \pi - \arccos\left(\frac{2}{3}\right)$, $\theta = \pi + \arccos\left(\frac{2}{3}\right)$
   (C) $\theta = \pi$, $\theta = \arccos\left(\frac{2}{3}\right)$, $\theta = \pi + \arccos\left(\frac{2}{3}\right)$
   (D) $\theta = \pi$, $\theta = \arccos\left(\frac{2}{3}\right)$, $\theta = 2\pi - \arccos\left(\frac{2}{3}\right)$

31. Find an expression for all solutions to the following trigonometric equation.
$$\sec^2\theta + \tan\theta = 1$$
   (A) $\theta = \pi k$ or $\theta = \frac{3\pi}{4} + \pi k$, where $k \in \mathbb{Z}$
   (B) $\theta = \frac{\pi}{2} + \pi k$ or $\theta = \frac{\pi}{4} + 2\pi k$, where $k \in \mathbb{Z}$
   (C) $\theta = 2\pi k$ or $\theta = \frac{3\pi}{4} + \pi k$, where $k \in \mathbb{Z}$
   (D) $\theta = \frac{\pi}{4} + \frac{\pi}{2}k$, where $k \in \mathbb{Z}$

32. Find all solutions in the interval $0 \le \theta < 4\pi$.
$$\cos\theta\tan\theta = 1$$

33. Select all of the following that are solutions to
$$\cot^2\theta + 4\csc\theta + 1 = 0$$
   (A) $\pi + \arcsin\left(\frac{1}{4}\right)$
   (B) $2\pi - \arcsin\left(\frac{1}{4}\right)$
   (C) 0
   (D) $\frac{\pi}{2}$
   (E) π
   (F) $-\arcsin(4)$
   (G) $\pi - \arcsin(4)$

34. Given $\tan\left(\frac{5\pi}{12}\right) = 2+\sqrt{3}$, find the value of $\cot\left(\frac{5\pi}{12}\right)$.
(A) $-2-\sqrt{3}$
(B) $2-\sqrt{3}$
(C) $3-\sqrt{2}$
(D) $3+\sqrt{2}$

35. The range of a certain function $f(x)$ is $[-2,10]$. If $g(x) = -3f(x+2)+5$, find the range of $g(x)$.
(A) $[-1,35]$
(B) $[-5,6]$
(C) $[-25,11]$
(D) $[-45,-9]$

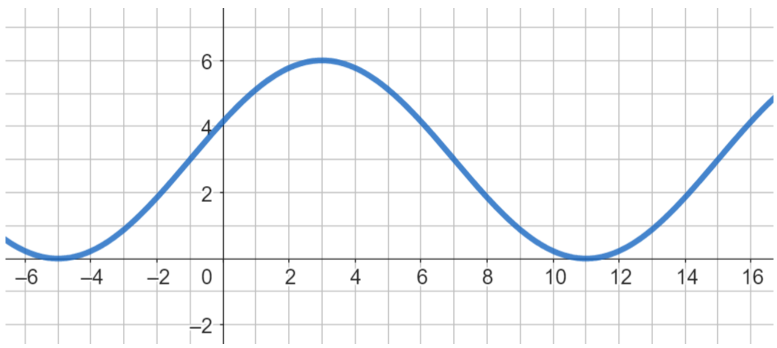
36. Find the range of $f(x) = 3\sec(2x) - 1$

37. Evaluate
a. $\arcsin\left(\frac{1}{2}\right)$
b. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$
c. $\arctan(0)$
d. $\arcsin(-1)$
e. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
f. $\arctan(\sqrt{3})$

38. Young sunflowers follow the sun's movements across the sky. In the morning, they face the rising sun in the east. Throughout the day, the head of the sunflowers rotates. By sunset, the sunflower is facing west. Throughout the night, the head of the sunflower reorients itself back toward facing east in anticipation of the rising sun. Depending on the orientation of the head of the flower, the height of the highest point on the flower changes. For one particular sunflower, at 6 AM the sunflower is at its minimum height of 52 inches. At noon the sunflower is at its maximum height of 60 inches. At 6 OM, the sunflower is back to its minimum height of 52 inches. Assume the height of the flower varies sinusoidally. Then which one of the following may best model $h(t)$, the height of the sunflower in inches t hours after midnight?
(A) $56+4\sin\left(\frac{\pi}{12}t\right)$
(B) $56+4\cos\left(\frac{\pi}{12}t\right)$
(C) $56+4\sin\left(\frac{\pi}{6}t\right)$
(D) $56+4\cos\left(\frac{\pi}{6}t\right)$

39. Which one of the following is an even function?
(A) $f(x) = \tan\left(x-\frac{\pi}{2}\right)$
(B) $f(x) = \cot\left(x-\frac{\pi}{2}\right)$
(C) $f(x) = \sin\left(x-\frac{\pi}{2}\right)$
(D) $f(x) = \cos\left(x-\frac{\pi}{2}\right)$

40. Three of the following are equivalent. One is not. Select the transformation that is not equivalent to the others.
   (A) The graph of $f(x)$ is dilated horizontally by a factor of $\frac{1}{2}$, then reflected over the y-axis, then translated horizontally 3 units.
   (B) The graph of $f(x)$ is reflected over the y-axis, then dilated horizontally by a factor of $\frac{1}{2}$, then translated horizontally 3 units.
   (C) The graph of $f(x)$ is translated horizontally -6 units, then reflected over the y-axis, then dilated horizontally by a factor of $\frac{1}{2}$.
   (D) The graph of $f(x)$ is dilated horizontally by a factor of $\frac{1}{2}$, then translated horizontally by 3 units, then reflected over the y-axis.



41. Which one of the following is an equation for the graph shown above?
   (A) $3\cos\left(\frac{\pi}{8}(x+1)\right)+3$
   (B) $3\sin\left(\frac{\pi}{8}(x+1)\right)+3$
   (C) $3\cos\left(\frac{\pi}{8}(x-1)\right)+3$
   (D) $3\sin\left(\frac{\pi}{8}(x-1)\right)+3$

42. Which one of the following is not a vertical asymptote on the graph $y=\tan\left(\frac{3}{2}x\right)$?
   (A) $-\frac{\pi}{3}$
   (B) $\frac{\pi}{3}$
   (C) $\frac{2\pi}{3}$
   (D) π

43. Find all values of θ in $0 \le \theta \le 2\pi$ that satisfy

$$2\cos^2\theta-\cos\theta-1<0$$

44. Let $f(x)=2x+1$, let $g(x)=4x$, and let $h(x)=(f\circ g)(x)$. Find the average rate of change of $h(x)$ over the interval $2 \le x \le 5$.

Questions #45-49 all pertain to the function $f(x) = \sin(x)$

45. On the interval $\left(\frac{\pi}{2}, \pi\right)$, $f(x)$ is...
    (A) Positive
    (B) Negative

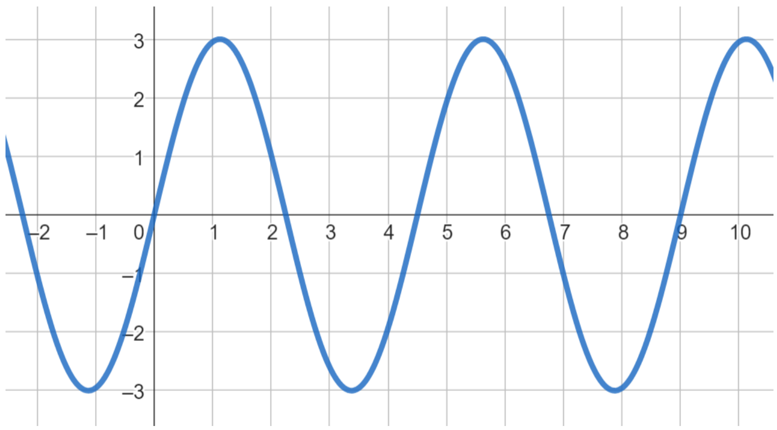
46. On the interval $\left(\frac{\pi}{2}, \pi\right)$, $f(x)$ is...
    (A) Increasing
    (B) Decreasing

47. On the interval $\left(\frac{\pi}{2}, \pi\right)$, the rate of change of $f(x)$ is...
    (A) Positive
    (B) Negative

48. On the interval $\left(\frac{\pi}{2}, \pi\right)$, the rate of change of $f(x)$ is...
    (A) Increasing
    (B) Decreasing

49. On the interval $\left(\frac{\pi}{2}, \pi\right)$, $f(x)$ is...
    (A) Concave up
    (B) Concave down

Below is the graph of $g(x)$. Use this graph to answer questions #50-52.



50. What is the period of $g(x)$? What is the amplitude of $g(x)$?

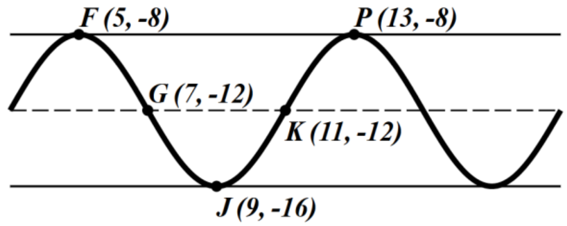
51. Let $h(x) = 2x - 4$. Find the period and amplitude of $(g \circ h)(x)$.

52. Let $h(x) = 2x - 4$. Find the period and amplitude of $(h \circ g)(x)$.

53. Consider the function $f(x) = \tan(x)$. Which one of the following statements correctly describes the behavior of this function on the interval $\left(0, \frac{\pi}{2}\right)$
    (A) This function is positive with a rate of change that is positive and decreasing.
    (B) This function is positive with a rate of change that is positive and increasing.
    (C) This function is negative with a rate of change that is positive and increasing.
    (D) This function is negative with a rate of change that is positive and decreasing.

54. Let $f(\theta) = 1 + 2\sin\theta$. Roughly sketch a graph of $y = f(\theta)$ to aid in answering this question. Which one of the following is an interval over which $f(\theta)$ is negative but has a positive rate of change?
    (A) $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$
    (B) $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right)$
    (C) $\left(\frac{7\pi}{6}, \frac{3\pi}{2}\right)$
    (D) $\left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

55. Refer to the graph of h shown below.

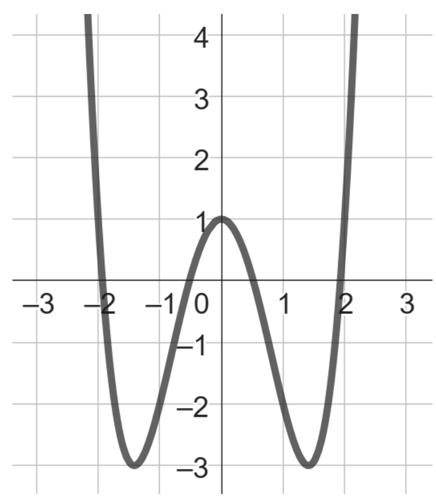


a. The function h can be written in the form $h(t) = a\sin\big(b(t+c)\big) + d$. Find values of the constants a, b, c, and d.
b. The t-coordinate of G is t_1. The t-coordinate of J is t_2. On the interval (t_1, t_2), which of the following is true about h?
   (A) h is positive and increasing.
   (B) h is positive and decreasing.
   (C) h is negative and increasing.
   (D) h is negative and decreasing.
c. The t-coordinate of G is t_1. The t-coordinate of J is t_2. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.
d. Find the average rate of change of h over the interval (t_1, t_2).
e. Use the average rate of change found in (d) to estimate $h(8.5)$.
f. Without the use of a calculator, will the value found in (e) overestimate or underestimate the actual value of $h(8.5)$? How do you know?

56. Let j be the function given by $j(x) = \frac{12}{\pi}\arctan(2x) + 4$. Find all zeros of the function j.

57. Let $f(x) = \frac{3x^3-1}{8}$.
   a. Find $f^{-1}(10)$
   b. Find an expression for $f^{-1}(x)$.

58. To the right is the graph of $y = g(x)$. Is $g(x)$ a one-to-one function?
   (A) Yes, $g(x)$ is one-to-one since every input value maps to a unique output value.
   (B) Yes, $g(x)$ is one-to-one since every output value is mapped from a unique input value.
   (C) No, $g(x)$ is not one-to-one since $g(-2) = g(2)$.
   (D) No, $g(x)$ is not one-to-one since $g(x)$ does not have origin symmetry.



59. Let $f(x) = 2x + 4$ and let $g(x) = \frac{x}{4}$. Find $(f^{-1} \circ g)(20)$

Some QR codes from throughout the semester:

Describing f vs. describing the rate of change of f

Polynomial End Behavior

Rational End Behavior

Trigonometry Ratios

Inverse Trig Values

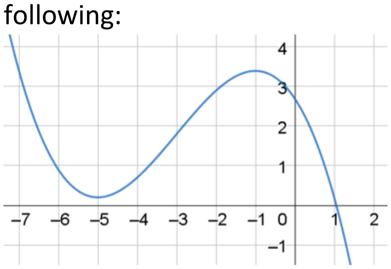
FRQ 3(C) Style Questions

Find the equation for a sinusoidal function

Holes vs. Vertical Asymptotes vs. x-Intercepts

AP Precalculus Midterm Review Answers

1.
   a. 3 (saying $x = 3$ or $x = 7$ would be incorrect as this question asked for the maximum, not where the maximum occurs)
   b. 0 (2 is a local minimum, but 0 is the global/absolute minimum)
   c. $x = 0$, $x = 5$, and $x = 8$
   d. $x = 3$ and $x = 7$
2. A (for the function to be concave down, slopes have to be decreasing)
3.
   a. If $2 - i$ is a zero of f, then $2 + i$ must be a zero as well. f is quartic, so there are four total zeros. Since at least two of the zeros are nonreal, there cannot be three real zeros. Yet the scenario states the graph of $f(x)$ has three x-intercpets. This is a contradiction.
   b. If $f(x)$ is an odd function with $f(4) = -3$, then $f(-4) = 3$. So, the average rate of change of $f(x)$ over the interval $[-4,4]$ is $\frac{f(4)-f(-4)}{(4)-(-4)} = \frac{-3-3}{8} = -\frac{6}{8} = -\frac{3}{4}$. Yet the scenario states that the average rate of change over that interval is $\frac{1}{2}$. This is a contradiction.
4. If $d(x)$ is cubic and $n(x)$ is quadratic, then he degree in the denominator of this rational function is more dominant than the degree of the numberator. The graph of this rational function must have $y = 0$ as a horizontal asymptote. This means $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$. Yet the scenario states $\lim_{x\to-\infty} f(x) = -\infty$. This is a contradiction.
5. Vertical asymptotes: $x = -2$ and $x = -\frac{1}{3}$
   Horizontal asymptote: $y = 2$
   Slant Asymptote: None
   Holes: None
   x-intercepts: $\left(-\frac{2}{3}, 0\right)$ and $(-1,0)$
   y-intercept: $(0, 2)$
   $\lim_{x\to-\infty} f(x) = 2$
6. Vertical asymptote: $x = 6$
   Horizontal Asymptote: None
   Slant Asymptote: None
   Holes: $(-1, 0)$
   x-intercept: $\left(\frac{3}{2}, 0\right)$
   y-intercept: $\left(0, -\frac{1}{2}\right)$
   $\lim_{x\to-\infty} f(x) = -\infty$
7. $y = -3x$
8. $(-\infty, 1] \cup (5, \infty)$
9. D
10. B
11. Answers may vary, but the function should have exactly one x-intercept and should vaguely resemble the following:

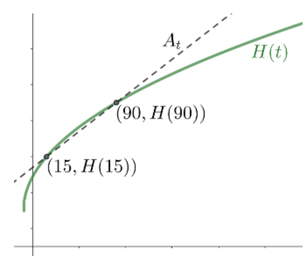


12.
   a. $\frac{f(2)-f(-2)}{(2)-(-2)} = \frac{(1)-(-4)}{(2)-(-2)} = \frac{5}{4}$
   b. $k = 8$

13.
   a. B
   b. $\lim_{x\to -2^+} f(x) = \infty$
   c. $\lim_{x\to 2} f(x) = 0$

14. B
15. C
16. A
17. D
18. 7
19. $r(x)$ has two vertical asymptotes, one hole, and no x-intercepts
20.
   a. Must be true. If $2+3i$ is a zero, then its complex conjugate $2-3i$ is a zero as well.
   b. Might be true. $2+3i$ and $2-3i$ are definitely zeros. If $-2+3i$ is a zero, then $-2-3i$ is also a zero. That is four nonreal zeros. That is possible for a fifth degree polynomial.
   c. Cannot be true. For a fifth degree polynomial with negative leading coefficient, $\lim_{x\to\infty} p(x) = -\infty$ while $\lim_{x\to -\infty} p(x) = \infty$.
   d. Might be true. $2+3i$ and $2-3i$ are definitely zeros. That leaves room for at most three real zeros. Perhaps $p(x)$ has a real zero of multiplicity 1 and a real zero of multiplicity 2.
   e. Cannot be true. If $\frac{1}{p(x)}$ has four vertical asymptotes, then $p(x)$ has four distinct real zeros. But $p(x)$ can only have at most 3 real zeros.

21.
   a. $\frac{H(90)-H(15)}{90-15} = \frac{63-60}{90-15} = \frac{3}{75} = \frac{1}{25} = 0.04$
   b. $\frac{H(45)-60}{45-15} \approx \frac{1}{25}$
   $H(45) - 60 \approx 1.2$
   $H(45) \approx 61.2$
   c. We are comparing the concave down function H to a value from the secant line passing through $\left(15, H(15)\right)$ and $\left(90, H(90)\right)$. For $15 < t < 90$, the graph of the secant line lies below the graph of $H(t)$ since H is concave down. So, at $t = 45$, the estimate based on average rate of change is less than the value of $H(45)$.



22.

a.	2^{nd} Quadrant	Reference Angle: $\frac{\pi}{5}$	Coterminal Angles: $-\frac{16\pi}{5}, \frac{4\pi}{5}$
b.	4^{th} Quadrant	Reference Angle: $\frac{\pi}{4}$	Coterminal Angles: $-\frac{\pi}{4}, \frac{15\pi}{4}$
c.	1^{st} Quadrant	Reference Angle: $\frac{\pi}{3}$	Coterminal Angles: $-\frac{5\pi}{3}, \frac{\pi}{3}$

23.
   a. $\frac{\sqrt{2}}{2}$
   b. 0
   c. -1
   d. -2
   e. Undefined
   f. $-\sqrt{3}$
   g. 1
   h. $-\frac{1}{2}$
   i. 0

24. $\sin\theta = -\frac{12}{13}$ $\quad\sec\theta = \frac{13}{5}$ $\quad\tan\theta = -\frac{12}{5}$
25. $\theta = \frac{5}{4}$
26. B

27. C
28. C
29. B
30. D
31. A
32. No solution!

$$\cos\theta \tan\theta = 1$$
$$\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) = 1$$
$$\sin\theta = 1$$

So it looks like $\frac{\pi}{2}$ and $\frac{5\pi}{2}$ would be solutions, right?

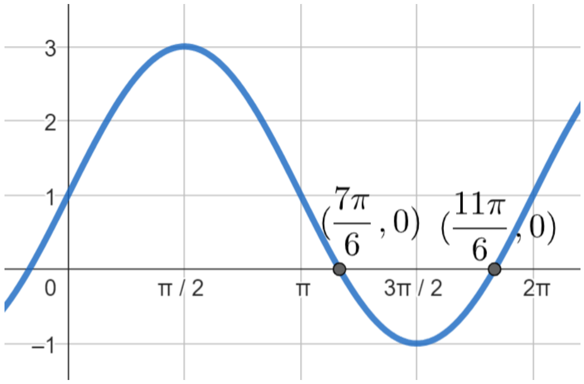
Wrong. $\tan\theta$ is undefined at those angles, so neither is a solution to the equation.

33. A and B
34. B
35. C
36. $(-\infty, -4] \cup [2, \infty)$

37.
    a. $\frac{\pi}{6}$
    b. $\frac{3\pi}{4}$
    c. 0
    d. $-\frac{\pi}{2}$
    e. $\frac{5\pi}{6}$
    f. $\frac{\pi}{3}$

38. D
39. C
40. D
41. B
42. C
43. $\left(0, \frac{2\pi}{3}\right) \cup \left(\frac{4\pi}{3}, 2\pi\right)$
44. $\frac{h(5)-h(2)}{(5)-(2)} = \frac{f(g(5))-f(g(2))}{5-2} = \frac{f(20)-f(8)}{5-2} = \frac{41-17}{5-2} = \frac{24}{3} = 8$
45. A
46. B
47. B
48. B
49. B
50. Period: $\frac{9}{2} = 4.5$ Amplitude: 3
51. Period: $\frac{9}{4}$ Amplitude: 3
52. Period: $\frac{9}{2}$ Amplitude: 6
53. B

54. D (consider the following graph)



55.

a. $a = 4$ $b = \frac{\pi}{4}$ $c = -11$ $d = -12$

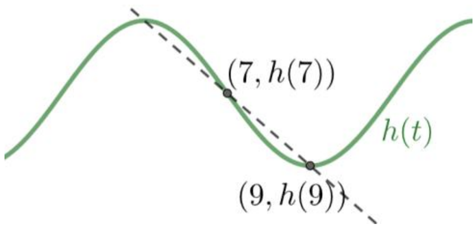
b. (D) h is negative and decreasing

c. On the interval (t_1, t_2), the graph of h is concave up. On this interval, the rate of change of h is increasing.

d. $\frac{h(t_2)-h(t_1)}{t_2-t_1} = \frac{(-16)-(-12)}{(9)-(7)} = \frac{-4}{2} = -2$

e. $h(t) \approx -12 + (-2)(t-7)$
$h(8.5) \approx -12 - 2(8.5-7) = -15$

f. Over the interval $7 < t < 9$, h is a concave up function. The estimate in part (e) was a value from the secant line passing through $(7, h(7))$ and $(9, h(9))$. Since the secant line is above the concave up graph (see picture below), its values overestimate the actual value of the concave up function. Part (e) overestimates the actual value of $h(8.5)$



56. $x = -\frac{\sqrt{3}}{2}$ is the only zero

57.

a. $f^{-1}(10) = 3$

b. $f^{-1}(x) = \sqrt[3]{\frac{8x+1}{3}}$

58. C

59. $\frac{1}{2}$